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Abstract

This paper examines the properties of exams and markets as alternative allocation devices under borrowing constraints. Exams dominate markets in terms of matching efficiency. Whether aggregate consumption is greater under exams than under markets depends on the power of the exam technology; for a sufficiently powerful test, exams dominate markets in terms of aggregate consumption as well. The positive effects of income taxation are analyzed and the optimal allocation scheme when wealth is observable is derived. The latter consists of a fellowship scheme in which markets set school prices but the government gives out fellowships based on need and the ability to obtain a given exam score.

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1. Introduction

The vast majority of goods and services in a capitalist economy are allocated solely by prices. As economists we think that we have a good understanding of why this is and should be the case. In a competitive economy with private goods, prices convey all the information needed by producers and consumers to ensure that scarce resources are allocated so that utility is maximized. The resulting allocation is Pareto efficient.

In contrast with the type of good described above, the provision of education is rather different. In many instances, and at all levels of education, prices are not the sole mechanism used to allocate students to schools. Grades, examination results, extracurricular activities, etc. may all play a role in guiding the admission decision. In fact, depending on the country and the institution, especially at the secondary and tertiary levels of education, both prices and exams play an important role in determining the allocation of students to schools. Prices can influence allocations directly, via a fee charged by a private school or university. They can also influence allocations indirectly, as is the case in the US where public secondary education is locally provided and, to a large extent, locally funded implying that it is necessary to purchase housing in a particular community to obtain a desired level of spending on education or a specific composition of peers.[1] Exams are also used as allocative devices, both on their own to determine admissions at selective public secondary schools or universities, and in conjunction with prices at many private schools and universities. The relative importance of prices and exams in guiding allocation decisions varies across countries. In many European and Latin American countries, for examples, prices for tertiary education are very low and exams are used to determine admissions; private universities exist but are often considered second rate. In Japan a one-shot entrance exam determines university admission and sorts students into their lifetime careers; rejection or acceptance by a prestigious university is seen as determining the student's entire future professional life. In the US, on the other hand, the majority of universities are private and both prices and exams play an important role in guiding the matching of students to universities.

There are many reasons to think that education is a special good and hence that a pure price mechanism may not do the best job of allocating students to schools. It has been argued, for example, that education may have significant

[1] Fernandez and Rogerson (1996,1998), Benabou (1993), and Epple and Romano (1998), among others, analyze the local provision of education in a model with borrowing constraints.

externalities, which may help explain why in all countries governments subsidize education and make some level of education mandatory. This does not in itself provide an explanation, however, for why exams are used to help guide the allocation decision. As long as the marginal return from investing in higher quality (or quantity) of education is greater for a higher ability student than for a lower one, one would expect that higher ability students would be willing to outspend the lower ability students to attend higher quality schools. In such a case one might expect to see students segregated by ability across schools, with prices providing the correct signals with respect to which school each should attend. Government intervention would be through subsidies to give incentives to low ability students to obtain education, or to encourage students of all abilities to attend school for a longer period of time than dictated strictly by their marginal private return.

Another argument regards education as a special good not because of the existence of social externalities, but rather because each student may have positive or negative externalities for other students or for the educational process. However, this externality (known as peer effects), also does not in and of itself justify the existence of exams as an allocative mechanism. In an economy in which ability is perfectly observable and with perfect capital markets, prices alone would be sufficient to determine optimally the allocation of students to a given distribution of schools. This would entail charging students of different abilities different prices (including negative ones, i.e. paying some students to attend a particular school).[2] If ability is not observable (the more plausible assumption), then exams that give a signal about ability should be a useful additional instrument in solving the assignment problem.[3] In such a case one might expect to see schools posting prices based on exam score (i.e. ability type) but not based on financial status. That is, students of the same ability should pay the same price at the same school or, to the extent that this is accomplished via fellowships, the latter should be based solely on merit.[4]

An important maintained assumption in the preceding discussion has been the existence of perfect capital markets. Borrowing constraints, however, are

[2] See Rothschild and White (1995) for an analysis of this problem.

[3] In general, if ability is unobservable (and in the absence of exams), a competitive equilibrium may not exist (see Rothschild and Stiglitz (1976)).

[4] It is interesting to note that this is not what we observe in the majority of cases, at least for undergraduate education in the United States. The majority of scholarships are based on financial need and are independent of merit (once a cutoff level-admissions-has been reached). This is far less the case though in graduate education, where fellowships are awarded mainly on merit.

often considered an important impediment to achieving a first-best allocation in a variety of environments.[5] This may be thought to be especially true in the case of education, where the unobservability of ability, moral hazard problems, and the fact that decisions are often made by parents rather than children, all serve to compound the problem of making loans against future human capital.[6]

It may be thought that in the presence of borrowing constraints, an education system that offers access to schools of different qualities based on exam scores may have some advantages over one based solely on fees. There are several objections, however, that may be raised to such a view. In the first place, exams are not perfect indicators of ability. They are influenced by prior expenditures that to some extent augment human capital and to some extent merely enhance test performance (e.g. Stanley Kaplan courses to improves SAT scores).[7] Wouldn't then the competition for higher test scores run into the same borrowing constraint problems as under market prices, as wealthy individuals outspend poorer ones in an attempt to obtain higher exams scores? Furthermore, if these expenditures are wasteful, wouldn't they impose additional costs on an economy that would not be borne under market prices? The objective of this paper is to contrast the relative benefits of exams versus market prices in a simple model that is able to capture at least some of these concerns.

The paper starts by establishing a benchmark of equilibrium under perfect capital markets, contrasting the allocation obtained under markets with that obtained by exams. To make the question interesting, we assume that individual ability and wealth are unobservable. Exam scores are assumed to be affected (positively) both by ability and by expenditures. These expenditures though are considered to be wasteful. That is, they do not in and of themselves increase utility nor output, but rather subtract from the resources available to an economy. We find that the matching of individuals to schools is the same under both

[5] Evans and Jovanovic (1989), for example, estimate that credit constraints are responsible for a total loss of investment by firms of 2.7 billion 1976 dollars annually and Pratap and Rendon (1997) estimate that over the period 1976-1995, the removal of liquidity constraints for a two digit industry could increase investment by an average of 34%.

[6] This is scarcely the first paper to note that borrowing constraints may play an important role in determining human capital accumulation. This is a feature that has been studied in different settings by Becker and Tomes (1979), and Loury (1981), and more recently by Fernandez and Rogerson (1998), Galor and Zeira (1993) and Glomm and Ravikumar (1992), among others. This is, though, to my knowledge the first paper to analyze the interaction of borrowing constraints and alternative allocation mechanisms such as exams, in the context of heterogeneous agents.

[7] Furthermore, it is often alleged that tests are culturally biased against some groups (racial, ethnic, gender, socio-economic, etc.).

mechanisms, but that markets achieve higher aggregate consumption due to the fact that exams involve wasteful expenditures.

Next the paper derives equilibrium under both allocation mechanisms under the assumption that individuals are unable to borrow. The interesting finding here is that exams always dominate markets with respect to matching efficiency, regardless of the exam's "power" to distinguish ability relative to expenditures.[8] Whether aggregate consumption is also higher under exams depends on the power of the latter; for a sufficiently powerful exam, aggregate consumption will indeed be higher as well. A subsequent section of the paper is devoted to an example that illustrates the properties of the two allocation mechanisms. The exam technology is parameterized such that the effect of different values of the "power" of an exam can be quantified.

The paper goes on to consider alternative policies (such as taxation) and identifies a mechanism that is able to achieve the first best when wealth is observable. This mechanism has the interesting feature of combining aspects of both market prices and exams. However, perhaps counter to what intuition might suggest, these exams are not very demanding; they do not require individuals to obtain high scores. A last section of the paper concludes and makes suggestions for future research.

2. A Simple Model

This objective of this paper is to contrast the properties of market prices relative to exams in a simple model.[9] To this end, we have chosen to work with a framework with two types of schools and a continuum of types of individuals. The distribution of schools and individuals is exogenous, and there are no peer effects or externalities.

We assume that there is a continuum of individuals, of measure one, characterized by their endowments of ability $a \epsilon [0, 1] \equiv I$ and wealth $w \epsilon [0, \overline{w}] \equiv \Omega$.

[8] Freeman (1996) also obtains the result that exams can improve efficiency in a framework with identical agents and borrowing constraints. The reason for this though is very different from the one offered here. In his model even an exam that is a pure lottery increases efficiency by allowing agents to specialize and take advantage of increasing returns to scale.

[9] A companion paper to this one is Fernandez and Gali (1997). That paper considers the case of a continuum of school types and a uniform distribution of agents. The analysis is more complicated then the one presented here and we are unable to obtain an analytically tractable example. Furthermore, it does not examine the effects of different interventions and allocation mechanisms such as income taxation and fellowships.

We assume that wealth and ability are unobservable, though nothing about how our two main mechanisms work depend on this.[10] The joint distribution of ability and wealth is given by $f(a, w)$, with $f(a, w) > 0$, $\forall a \epsilon I$, $\forall w \epsilon \Omega$. The cumulative distribution $F(a', w') = \int_0^{a'} \int_0^{w'} f(a, w)\ dwda$ is assumed to be continuous, with $F(a, 0) = 0$ and $F(1, \overline{w}) = 1$. Schools are in fixed supply and are distinguished by their quality, s, which can be either high (H) or low (L). There is a measure one of schools and a fraction $1-\alpha$ of them are of high quality. We assume that a measure μ of schools is able to accommodate exactly a measure μ of individuals (hence the high quality schools can accommodate a fraction $1 - \alpha$ of the population).

An individual with ability a who attends school s, $s \in \mathrm{S} = \{L, H\}$, obtains $X(a, s)$, where X can be thought of as a production function or as a human capital function (with the price of a unit of human capital or of the output generated normalized to equal one). We assume that agents have strictly increasing utility functions over consumption and that their output from not attending any school is normalized to zero. Furthermore, we assume X is continuous, bounded, with $X_a \geq 0$, $X_s \geq 0$ (i.e., output or human capital is increasing both in ability and schooling), and $X_{as} > 0$, i.e., ability and schooling are complements. This last assumption plays a key role in the characterization of efficient allocations. A last assumption that facilitates exposition is $X(0, L) = 0$, (i.e., the lowest ability agent obtains a payoff of zero when she attends the lower-quality school).[11]

Independently of the allocation mechanism and borrowing environment, the actions of agents can be thought of as occurring within the following two-stage framework. In the first stage, individuals incur their desired expenditures (and desired borrowing as well if capital markets are operating) and are allocated, according to the assignment rules, to a school $S(a, w)$ ($S : I\mathrm{x}\Omega \rightarrow \mathrm{S}$). In the second stage they generate output X, repay loans (if any), and consume their output plus initial wealth minus debt repayment and first-stage expenditures.

2.1. The Efficient Allocation

Before comparing allocation mechanisms, it is useful to review the efficient allocation in this framework. The assumptions on technology (namely $X_{as} > 0$) render the matching problem in our model identical to Becker's (1973) marriage

[10] Observability of ability renders the mechanism design problem unintersting since a social planner could simply mandate which individuals should attend which schools. Section 5 examines a fellowship scheme under the assumption that wealth is observable.

[11] As will be clear further on, this assumption allows us to get rid of price equilibria under perfect capital markets that differ from each other solely by a constant.

problem.[12] In such a framework the efficient allocation is characterized by positive assortative matching, i.e., the "highest quality" female matching with the "highest quality" male, the second-best female with the second-best male, etc. In our two-school and continuum-of-agent-types model, the efficient allocation is instead characterized by a partition of the ability space, a^*, such that all individuals with $a > a^*$ are assigned to H and all those with $a < a^*$ are assigned to L, independently of their wealth. The cutoff level a^* exactly exhausts the capacity of the schools, i.e., $F(a^*, \overline{w}) = \alpha$.

The above result follows entirely from the assumption of complementarity between a and s, i.e. from $X_{as} > 0$. To see why the above allocation is efficient, note that if there were a positive measure of agents with $a > a^*$ assigned to $s = L$, then for every pair of agents (a', w'), (a'', w'') with school assignments $S(a', w') = H$ and $S(a'', w'') = L$ such that $a'' > a'$, it would be possible to rematch these agents, i.e. match a'' to H and a' to L. This would increase total output since $X_{as} > 0$ implies that the gains from rematching the higher-ability agent are greater than the losses from rematching the lower-ability agent, i.e.:

$$X(a'', H) - X(a'', L) > X(a', H) - X(a', L) \quad \forall a'' > a' \tag{2.1}$$

It will be useful to note the aggregate output obtained in the efficient allocation for purposes of future comparisons. It is given by:

$$Y^* = \int_{a^*}^{1} \int_{0}^{\overline{w}} X(a, H) f(a, w) dw da + \int_{0}^{a^*} \int_{0}^{\overline{w}} X(a, L) f(a, w) dw da \tag{2.2}$$

2.2. Two Allocation Mechanisms

What are the assignment rules by which different mechanisms allocate individuals? We will consider two different environments with two different assignment rules. In the first case we will assume a private market for schools. Markets, it is clear, will allocate individuals to schools based on their willingness and ability to pay a school's price. Second, we will assume public ownership of schools and exams that allocate individuals to schools based on their performance. The exam assignment rule will assign those individuals who obtain test scores over some cutoff level

[12] Thus, our paper can also be seen as contributing to a relatively small literature that examines matching problems, including Cole, Mailath, and Postlewaite (1992), Acemoglu (1995), Kremer and Maskin (1996), Legros and Newman, and Burdett and Coles (1996), by analyzing the effect of borrowing constraints. See Sattinger (1993) for a review of assignment models.

to H, and all others to L. Thus an exam mechanism will allocate individuals according to their ability and willingness to obtain a given test score.

Before comparing some of the general properties of the two allocation mechanisms, we need to present additional features of exams. The exam is a technology V whose outcome (the score)-v-depends both on the ability and expenditures, e, made by the agent taking the exam. Thus, $v = V(a, e)$ and we assume that $V_a > 0$, $V_e > 0$ (i.e. the score is increasing in ability and expenditures), and that $V(a, 0) = 0$, $\forall a \in I$ (i.e. expenditures are necessary to obtain a positive score).[13] These expenditures can be thought of as being incurred directly in exam preparation (tutors, etc.) or indirectly via the choice of high school attended (if these are colleges).

It is often convenient to work with the dual of V, namely $E(a, v)$, which gives the expenditures needed by an agent of ability a to obtain a score of v. Note that our assumptions on V imply $E_a < 0$ and $E_v > 0$. For reasons that will become clear soon, we assume $E_{va} \leq 0$ (that is, the change in expenditures needed to obtain a marginal increase in a given score is non-increasing in a).

The objective of this section is to show that both mechanisms induce similar preferences over outcomes on the part of individuals. Take markets first: Figure 1 depicts the indifference curves of two individuals through a given school quality-price bundle, $(\overline{s}, \overline{p})$. As can be shown by finding the slope of the indifference curve $(u = \overline{u})$ through a given point and differentiating it with respect to a, higher-ability individuals have steeper indifference curves in this space, i.e.,

$$\frac{\partial}{\partial a}\left(\frac{dp}{ds}\mid_{u=\overline{u}}\right) = X_{as}(a, s) > 0 \tag{2.3}$$

This holds independently of the bundle and of the ability levels considered implying, as shown in Figure 1, that indifference curves are single crossing.

Figure 2 performs a similar exercise for the case of exams. The relevant space, in this case, is school quality and exam scores (s, v). Differentiation of the slope of an indifference curve with respect to ability yields:

$$\frac{\partial}{\partial a}\left(\frac{dv}{ds}\mid_{u=\overline{u}}\right) = \frac{X_{as}E_v - E_{va}X_s}{E_v^2} > 0 \tag{2.4}$$

That is, the indifference curves of individuals with different abilities are single crossing; individuals with higher ability have steeper indifference curves through

[13] This last assumption is not essential; it is made to facilitate characterization of equilibrium.

any given point than individuals with lower ability.[14]

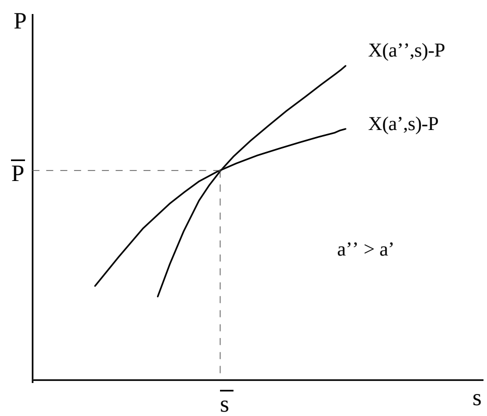

Figure 1

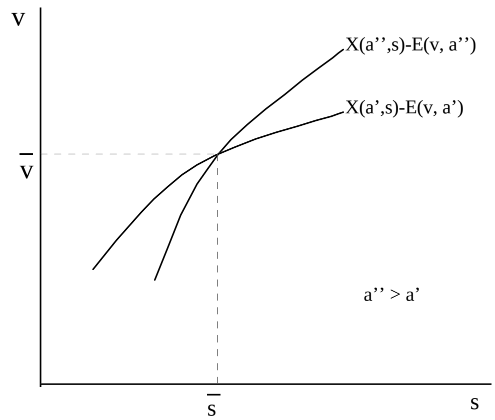

Figure 2

The above single-crossing results have powerful implications for the properties of equilibrium both with and without perfect capital markets. Under perfect capital markets, equations (2.3) and (2.4) imply that individuals with greater ability will always be willing (and able) to outspend or outscore lower ability individuals to ensure that they are allocated to H. That is, under perfect capital markets,

$$S(a'', w'') \geq S(a', w');\ \forall a'' > a', \forall w \tag{2.5}$$

If individuals cannot borrow, the results obtained in (2.5) no longer hold since although higher-ability individuals desire to outspend or outscore lower-ability individuals, their lack of resources may not permit them to do so. Hence, in the presence of borrowing constraints, (2.5) is modified to:

$$S(a'', w'') \geq S(a', w');\ \forall a'' > a' \text{ and } \forall w'' \geq w' \tag{2.6}$$

under markets (indicating that individuals of greater ability will always be allocated to at least as high an s if they are at least as wealthy). With exams, the corresponding implication is:

[14] Note that the assumption $E_{va} \leq 0$ guarantees steeper indifference curves for higher-ability individuals (given our assumptions on production technology).

$$S(a'', w'') \geq S(a', w');\ \forall a'' > a' \text{ and } \forall w', w'' \text{ such that } V(a'', w'') \geq V(a', w') \tag{2.7}$$

i.e., individuals of higher ability will be allocated to at least as high an s if they can afford to generate at least the same score.

3. Equilibrium under Perfect Capital Markets

This section assumes that agents have access to perfect capital markets. In order to avoid the (unenlightening in this case) complications that accompany endogenizing the market for loans, we assume that this is an external capital market that operates at a constant and riskless interest rate which without loss of generality is normalized to zero. We next turn to characterizing equilibrium under both allocation mechanisms.

3.1. Markets

An equilibrium is (i) a price for each school such that demand for schools of each type equals supply, (ii) schools maximize profits (i.e. given that the distribution of schools is in fixed supply, in this context they simply charge the greatest price they can), and (iii) individuals maximize utility.

Given that schools are infinitesimally small and hence perfectly competitive, schools of the same quality must charge the same price in equilibrium. An implication of the discussion in the preceding section is that with perfect capital markets, higher-ability agents are willing and able to outspend lower ability agents in order to attend a higher quality school. Thus, the agents that attend L must include those with ability zero. Consequently, our assumptions of $X(0, L) = 0$, $f(a, w) > 0$, and continuity of X with respect to a imply that the price of a low-quality school, P_L, equals zero.

Single-crossing implies that equilibrium can be represented by an ability and high-quality school price (P_H) pair $(a_M^\star, P_H^*)$ such that all individuals with ability lower than a_M^* attend L and those with higher ability attend H. Utility maximization implies that all individuals that attend H must be at least as well off as by attending L (and vice versa for those individuals that attend L.) Hence, for all (a, w) such that $S(a, w) = H$, given $P_L = 0$, we must have $\Delta(a) \geq P_H^*$, where

$$\Delta(a) \equiv X(a, H) - X(a, L) \tag{3.1}$$

(with the reverse inequality must hold for those who attend L).[15] Furthermore, the fraction of individuals that attend H must equal $1-\alpha$. This, and continuity of X with respect to a, yields the following two equilibrium conditions for $(a^{\star}_M, P^*_H)$:

$$\Delta(a^{\star}_M) = P^*_H \tag{3.2}$$

$$\int_{a^*_M}^{1} \int_{0}^{\overline{w}} f(a,w)\ dw\ da = 1-\alpha \tag{3.3}$$

with single-crossing implying:

$$S(a,w) = \begin{cases} H,\ \forall a \geq a^*_M \\ L,\ otherwise \end{cases} \tag{3.4}$$

It should be noted that P^*_H exists and is unique since the fraction of individuals for whom $\Delta(a) \geq P_H$ is continuous and decreasing in P_H (with a fraction of one for whom $\Delta(a) \geq P_H = 0$ and a fraction of zero for whom $\Delta(a) \geq P_H = \Delta(1)$). See Figure 3 for a depiction of equilibrium.

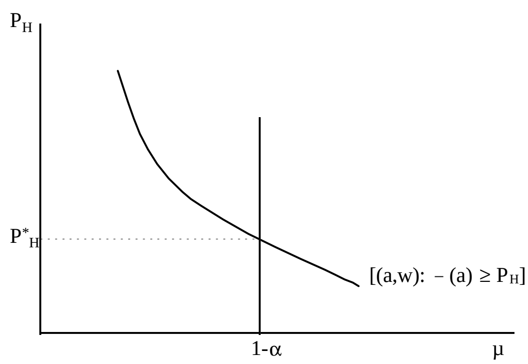

Figure 3

3.2. Exams

The assignment mechanism under exams is a rule that states a cutoff score level v^* such that all individuals with $v \geq v^*$ attend H and all others attend L. Since schools in this environment are completely passive and do not obtain any revenue

[15] Note that single-crossing implies $\Delta(a)$ increasing in a.

(i.e., do not charge a price), it is best to think of them as being owned by the state, with the government setting v^* such that $1-\alpha$ agents obtain at least this score. Note that since higher scores mean greater expenditures, no agent will in equilibrium obtain a score greater than $v^\star$. Furthermore, those agents who in equilibrium attend L will not expend any resources in the generation of scores. Hence, an equilibrium is (i) a score v^* such that a fraction $1-\alpha$ of agents obtain that score, and (ii) individuals maximize utility.

Again, single crossing implies that equilibrium can be characterized by a pair (a_T^*, v^*) such that all individuals with ability greater than a_T^* obtain the score v^* and attend H. Utility maximization implies that these individuals must be at least as well off as by making zero expenditures and attending L. Hence, for these individuals it must be the case that $\Delta(a) \geq E(v^\star, a)$ (with the opposite inequality holding for those agents that attend L). Consequently, equilibrium conditions on $(a_T^\star, v^\star)$ require:

$$\Delta(a_T^\star) = E(a_T^\star, v^\star) \tag{3.5}$$

$$\int_{a_T^*}^{1} \int_0^{\overline{w}} f(a,w)\; dw\; da = 1-\alpha \tag{3.6}$$

with single-crossing implying:

$$S(a,w) = \begin{cases} H,\ \forall a \geq a_T^* \\ L,\ otherwise \end{cases} \tag{3.7}$$

Note that $(a_T^\star, v^\star)$ exists and is unique since the fraction of agents with $\Delta(a) \geq E(a,v)$ is continuous and decreasing in v (with a fraction of one such that $\Delta(a) \geq E(a,0)$ and a fraction of zero for whom $\Delta(a) \geq E(a,\overline{v})$, where $\overline{v}$ is defined by $E(1,\overline{v}) = \Delta(1)$). A figure similar to Figure 3 can be used to depict equilibrium.

3.3. Markets vs Exams

We next turn to a comparison of markets and exams under perfect capital markets. In order to do this, we need to discuss how expenditures made under exams compare with fees charged by schools under markets. We assume that expenditures to improve exam scores are completely wasteful in the sense that they do not in and of themselves contribute to consumption or utility. Hence one can think of those expenditures as withdrawing resources (say tutors for SAT exams) from other productive potential uses in the economy, without in and of themselves increasing output.[16] Fees charged for schools under markets, on the other hand, are simply a transfer from one set of individuals to another (the owners of schools–presumably all agents in the economy). Although without lump-sum transfers among agents we cannot speak of one mechanism Pareto dominating another, for the sake of comparison we evaluate two features: (i) Matching efficiency (i.e. aggregate output), and (ii) Aggregate consumption.

The first thing to note, comparing equations (3.3) and (3.6), is that both markets and exams achieve the same final allocation, i.e. the same individuals are matched with the same schools and that this is the efficient allocation characterized in section 2.1. Hence (using M and T to indicate variables under markets and tests, respectively, and * to indicate perfect capital markets), we have:

$$a_M^\star = a_T^\star = a^* \tag{3.8}$$

implying that aggregate output is likewise the same, i.e.:

$$Y_M^* = Y_T^* = Y^* \tag{3.9}$$

Aggregate consumption (simply aggregate output plus initial aggregate wealth $W \equiv \int_0^1 \int_0^{\overline{w}} wf(a,w)\, dwda$ minus any waste), on the other hand, differs as a result of the sum of wasteful expenditures, ε^*, with exams. Hence,

$$C_M^* = Y_M^* + W = C^* \tag{3.10}$$

but

$$C_T^* = Y_T^* + W - \varepsilon^* = C_M^* - \varepsilon^* \tag{3.11}$$

where

$$\varepsilon^* = \int_{a^*}^1 \int_0^{\overline{w}} E(a, v^\star) f(a,w)\, dwda \tag{3.12}$$

[16]It might well be argued that going to, say, a better high school, produces greater human capital as well as enhancing exam taking technique, so that both output and the exam technology are affected by greater expenditures, i.e. $X(a + h(a,e), s)$ and $v = V(a + h(a,e), e)$ where h is the additional human capital produced by an agent of ability a and expenditures of e. As long as both under markets and exams we still obtain single-crossing indifference curves (which under reasonable assumptions for h we do), then the same sorting implications of 2.5 and 2.7 obtain. Thus, by considering all expenditures as wasteful, we are only making the case for exams harder.

Lastly, note that equations (3.8), (3.2) and (3.5) imply that

$$P_H^* = E(a^\star, v^\star) \tag{3.13}$$

i.e. the price of the high quality school under markets equals the exam expenditures of the lowest ability agent to attend H in equilibrium.

The results obtained above imply that the matches achieved under both exams and markets are the same as the efficient allocation. Not surprisingly, with perfect capital markets the price mechanism is able to achieve the first best level of output and hence also maximizes aggregate consumption (i.e. obtains C^*). The ability of exams to achieve matching efficiency is due to the assumptions on technology; the exam technology does not distort higher-ability agents' willingness to outscore lower-ability ones, and perfect capital markets make it feasible for them to do so. The lower aggregate consumption achieved under exams follows immediately from our assumption of exam expenditures being wasteful.

4. Equilibrium under Borrowing Constraints

In this section we derive equilibrium under markets and exams assuming that there is no recourse to borrowing at all. We do not model here the reason behind the market failure, though, for example, inability to penalize recalcitrant borrowers or an unverifiable output level would be sufficient to close down capital markets.

4.1. Markets

To begin with, note again that perfect competition, single crossing, $X(0, L) = 0$, X continuous in a, and $f(a, w) > 0\ \forall a \epsilon I, w \epsilon \Omega$, imply that the equilibrium price of a low-quality school is zero. Thus, an equilibrium is now a pair $(\widehat{a}_M, \widehat{P}_H)$ such that (i) individuals maximize their utility subject to a no-borrowing constraint, (ii) schools maximize profits, and (iii) demand for schools of each type equals their supply.

Consequently, $(\widehat{a}_M, \widehat{P}_H)$ must satisfy:

$$\Delta(\widehat{a}_M) = \widehat{P}_H \tag{4.1}$$

$$\int_{\widehat{P}_H}^{\overline{w}} \int_{\widehat{a}_M}^{1} f(a, w)\ da\ dw = 1 - \alpha \tag{4.2}$$

with single crossing implying

$$S(a,w) = \begin{cases} H, \forall a \geq \widehat{a}_M \text{ and } w \geq \widehat{P}_H \\ L, \; otherwise \end{cases} \tag{4.3}$$

Note that the lower limit of the wealth integral in equation (4.2) is no longer zero as under perfect capital markets but is instead $\widehat{P}_H$, reflecting the inability of agents to borrow. Equilibrium can now be depicted as in Figure 4 with all the agents in the shaded area attending H and all others attending L.

Aggregate output and consumption are now given by:

$$\begin{aligned} \widehat{Y}_M \;\; = \;\; & \int_{\widehat{a}_M}^{1} \int_{\widehat{P}_H}^{\overline{w}} X(a,H) f(a,w) \; dw \; da \; + \\ & \int_{0}^{\widehat{a}_M} \int_{\widehat{P}_H}^{\overline{w}} X(a,L) f(a,w) \; dw \; da + \int_{\widehat{a}_M}^{1} \int_{0}^{\widehat{P}_H} X(a,L) f(a,w) dw \; da \end{aligned} \tag{4.4}$$

with $\widehat{C}_M = \widehat{Y}_M + W$.

Uniqueness and existence of equilibrium can be shown using the same argument as under perfect capital markets except that the demand for high-quality schooling as a function of P_H is given by the fraction of agents with $\min(\Delta(a), w) \geq P_H$.

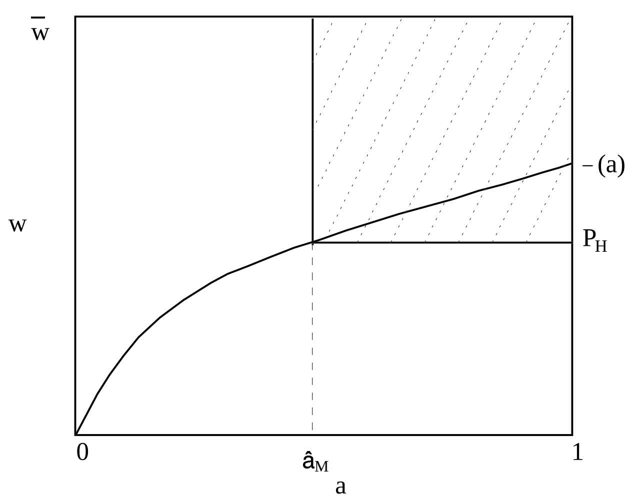

Figure 4

4.1.1. A Comparison of Markets With and Without Borrowing Constraints

It is of interest to compare the functioning of markets with and without borrowing constraints. A first result is that the lowest ability individual to attend H in equilibrium is higher under perfect capital markets than under borrowing constraints, i.e.,

Theorem 4.1. $\widehat{a}_M < a^\star$.

Proof: Let $H(a)$ be defined by

$$H(a) \equiv \int_{\Delta(a)}^{\overline{w}} \int_a^1 f(a,w)\ da\ dw$$

that is, $H(a)$ is the fraction of individuals with ability greater than a and wealth greater than $\Delta(a)$. Note that $H'(a) < 0$. With borrowing constraints, equations (4.1) and (4.2) imply that $1-\alpha = \int_{\Delta(\widehat{a})}^{\overline{w}} \int_{\widehat{a}_M}^1 f(a,w)\ da\ dw = H(\widehat{a}_M)$. With perfect capital markets, on the other hand, we have $1-\alpha = \int_0^{\overline{w}} \int_{a^\star}^1 f(a,w) dadw > H(a^\star)$. Hence, $H(\widehat{a}_M) > H(a^*)$, implying $\widehat{a}_M < a^\star$.||

It is now easy to show that the price of high-quality schooling is higher under perfect capital markets than under borrowing constraints.

Corollary 4.2. $P_H^* > \widehat{P}_H$.

Proof: This follows directly from Theorem (4.1) and equations (3.2) and (4.6).||

Neither of the results above are surprising: Borrowing constraints require a lower price for the high-quality school in order to fill capacity in both types of schools. But this decreased price makes the high-quality school more attractive to lower ability agents, decreasing the ability associated with the "marginal" agent that attends H relative to perfect capital markets.

Of course, the change in allocation implies that output (and hence consumption) is also lower under borrowing constraints relative to perfect capital markets, i.e.,

$$\widehat{Y}_M + W = \widehat{C}_M < Y^* + W = C^* \tag{4.5}$$

4.2. Exams

With borrowing constraints, an equilibrium under exams is an $(\widehat{a}_T, \widehat{v})$ pair such that (i) individuals maximize utility subject to a no-borrowing constraint and (ii) a fraction $1-\alpha$ of agents obtain a score of $\widehat{v}$. Thus, $(\widehat{a}_T, \widehat{v})$ must satisfy:

$$\Delta(\widehat{a}_T) = E(\widehat{a}_T, \widehat{v}) \tag{4.6}$$

$$\int_{\widehat{a}_T}^{1} \int_{E(a,\hat{v})}^{\overline{w}} f(a,w)\ dw\ da = 1-\alpha \tag{4.7}$$

with

$$S(a,w) = \begin{cases} H,\ \forall a \geq \widehat{a}_T \text{ and } w \geq E(\widehat{a}_T, \widehat{v}) \\ L,\ otherwise \end{cases} \tag{4.8}$$

Aggregate output and consumption are now given by:

$$\begin{aligned} \widehat{Y}_T &= \int_{\widehat{a}_T}^{1} \int_{E(a,\widehat{v})}^{\overline{w}} X(a,H) f(a,w) dwda + \int_{\widehat{a}_T}^{1} \int_{0}^{\overline{w}} X(a,L) f(a,w) dwda \\ &+ \int_{\widehat{a}_T}^{1} \int_{0}^{E(a,\widehat{v})} X(a,L) f(a,w)\ dwda \end{aligned} \tag{4.9}$$

and

$$\widehat{C}_T = \widehat{Y}_T + W - \widehat{\varepsilon} \tag{4.10}$$

where

$$\widehat{\varepsilon} = \int_{\widehat{a}_T}^{1} \int_{E(a,\widehat{v})}^{\overline{w}} E(a,\widehat{v})\ dwda \tag{4.11}$$

Existence and uniqueness of equilibrium can be proved as under perfect capital markets, with the demand for H as a function of v now given by the fraction of individuals with $\min(\Delta(a), w) \geq E(a,v)$. Equilibrium is depicted in Figure 5. The shaded area represents the set of agents that in equilibrium attend H.

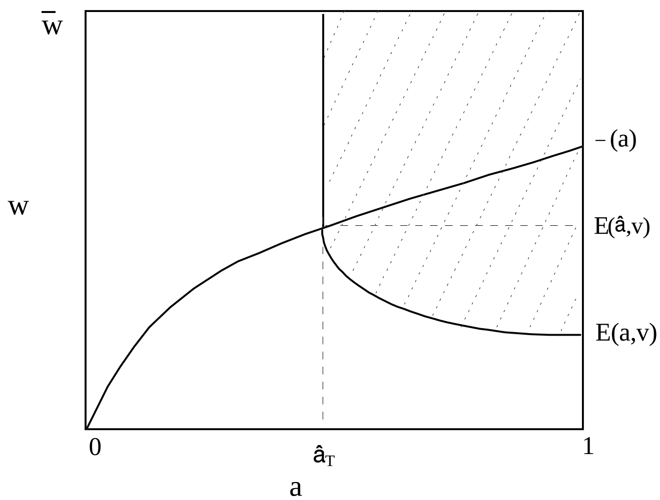

Figure 5

4.2.1. A Comparison of Exams With and Without Borrowing Constraints

As with markets, it is easy to show that the effect of borrowing constraints is to allow lower-ability level agents to attend H, and to decrease the score associated with the latter.

Theorem 4.3. $\widehat{a}_T < a^\star$.

Proof: Noting that $\widehat{v} > 0$, we have $1 - \alpha = \int_{a^\star}^{1} \int_{0}^{\overline{w}} f(a,w)\ dw\ da >$ $\int_{a^\star}^{1} \int_{E(a,\widehat{v})}^{\overline{w}} f(a,w)\ dw\ da$. Hence, $a^\star > \widehat{a}_T$.||

Corollary 4.4. $\widehat{v} < v^\star$.

Proof: Follows immediately from Theorem 4.3 and equations (3.5) and (4.6).||

As with our comparison of markets with and without borrowing constraints, obtaining a different allocation with borrowing constraints than that obtained under perfect capital markets implies

$$Y^* > \widehat{Y}_T \tag{4.12}$$

This does not allow us to conclude, however, that a necessary implication is $C_T^* > \widehat{C}_T$, since if wasteful expenditure under perfect capital markets is sufficiently larger than that under borrowing constraints, aggregate consumption can actually be greater under the latter.

Next we turn to a comparison of markets and exams under borrowing constraints.

4.3. Markets vs Exams

In this section we establish our main results regarding the benefits of exams relative to markets. We start out by showing that the lowest-ability individual to attend H under exams is of higher ability than the lowest-ability individual to attend H under markets.

Theorem 4.5. $\widehat{a}_T > \widehat{a}_M$.

Proof: Suppose not and instead suppose $\widehat{a}_T \leq \widehat{a}_M$. By equations (4.6) and (4.1), this implies $\Delta(\widehat{a}_T) \leq \Delta(\widehat{a}_M)$ and hence $E(\widehat{a}_T, \widehat{v}) \leq \widehat{P}_H$. But, $1 - \alpha = \int_{\widehat{a}_T}^{1} \int_{E(a,\widehat{v})}^{\overline{w}} f(a, w)dwda > \int_{\widehat{a}_T}^{1} \int_{E(\widehat{a}_T,\widehat{v})}^{\overline{w}} f(a, w)dwda$ (given $E_a < 0$) $\geq \int_{\widehat{a}_M}^{1} \int_{\widehat{P}_H}^{\overline{w}} f(a, w)dwda = 1 - \alpha$ (where the weak inequality follows from $\widehat{a}_M \geq \widehat{a}_T$ and $\widehat{P}_H \geq E(\widehat{a}_T, \widehat{v})$), a contradiction.||

Corollary 4.6. $E(\widehat{a}_T, \widehat{v}) > \widehat{P}_H$.

Proof: Follows immediately from the previous theorem and equations (4.6) and (4.1).||

An important implication of the preceding results is that under borrowing constraints exams possess greater matching efficiency than markets. This is established first using Figure 6 and then algebraically in Theorem 4.7 below.

Figure 6 allows us to compare the school assignments under exams and markets. Note that the area contained to the North-East of ABC represents the set of agents that attend H under exams and the area $DE\widehat{P}_H$ does the same for markets. The complements of each of these areas represents the set of agents that attend L under exams and markets respectively. Thus all agents in the shaded areas have the same assignments under both mechanisms. The agents in $ABDEF$ (hereafter area 2), however, attend H under markets and L under exams, whereas the agents represented by the area $FC\widehat{P}_H$ (hereafter area 1) attend L under markets and H under exams. Given the fixed capacity of the school system, the fraction of agents represented by area 1 must equal that represented by area 2. But, then, $X_{as} > 0$ implies that the net gain from reallocating higher ability agents to higher-quality schools in exchange for lower ability agents to lower-quality schools is positive, i.e., $\int\int_1 (X(a,H) - X(a,L))f(a,w)dwda > \int\int_2 (X(a,H) - X(a,L))f(a,w)\, dwda$.

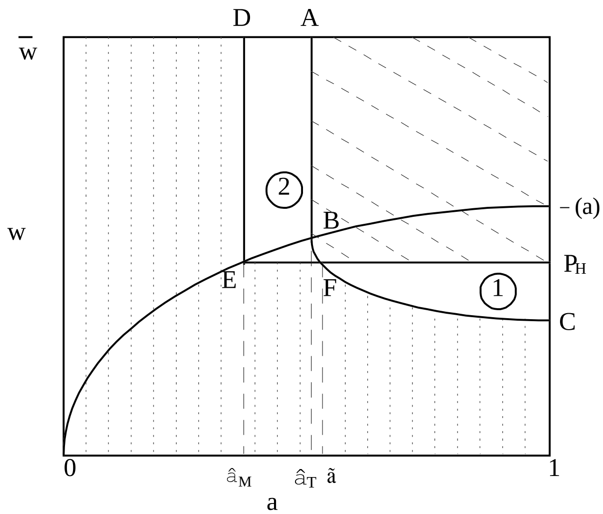

Figure 6

Theorem 4.7. $\widehat{Y}_T > \widehat{Y}_M$.

Proof: We begin by defining $\tilde{a}$ implicitly by $E(\tilde{a}, \widehat{v}) = \widehat{P}_H$. Thus, making use of Figure 6, we have $\widehat{Y}_T - \widehat{Y}_M = \int_{\tilde{a}}^{1} \int_{E(a,\widehat{v})}^{\hat{P}_H} (X(a,H) - X(a,L))f(a,w)dwda - \int_{\widehat{a}_M}^{\widehat{a}_T} \int_{\hat{P}_H}^{\overline{w}} (X(a,H) - X(a,L))f(a,w)dwda - \int_{\widehat{a}_T}^{\widetilde{a}} \int_{P_H}^{E(a,\widehat{v})} (X(a,H) - X(a,L))f(a,w)dwda > \Delta(\widetilde{a})\{\int_{\widetilde{a}}^{1} \int_{E(a,\widehat{v})}^{\hat{P}_H} f(a,w)dwda - \int_{\widehat{a}_M}^{\widehat{a}_T} \int_{\hat{P}_H}^{\overline{w}} f(a,w)dwda - \int_{\widehat{a}_T}^{\widetilde{a}} \int_{P_H}^{E(a,\widehat{v})} f(a,w)dwda\} = \Delta(\widetilde{a}).0 = 0.$ $||$

It is important to understand that, contrary to what intuition may suggest, the greater aggregate output obtained under exams does not result from the "marginal price" of the high-quality school being lower under exams than under markets. Indeed, as Corollary 4.6 establishes, the opposite is true, i.e., $E(\widehat{a}_T, \widehat{v}) > \widehat{P}_H$. Rather, the more efficient matching is due to the lower effective price for sufficiently high ability individuals, i.e., to the properties of the test technology, which make it less expensive for higher ability individuals to obtain a given score and more expensive for lower ability agents to do so. Exams make it possible for individuals to use another attribute–ability–to compete, which serves to effectively loosen borrowing constraints for high ability individuals and tighten them for low ability individuals.

Of course, a comparison of matching or productive efficiency of two allocation mechanisms is not as interesting as a comparison of the consumption they permit, since the magnitude of the resources expended in achieving the final allocation matters. In fact, it is not true that exams necessarily dominate markets in terms of aggregate consumption. In general, this will depend upon the "power" of the testing technology, the production function, and the income and ability distribution of agents. Below we establish a limiting result that guarantees that a sufficiently "powerful" testing technology (in a sense that will be made rigorous) implies that exams will dominate markets in terms of aggregate consumption as well.

Theorem 4.8. *Given a test technology that can be written as* $E(a, v) = h(v)g(a)^{-\lambda}$ *with* $h(0) = 0, h' > 0$ *and* $g, g' > 0$, *we can define a family of technologies,* E_λ, *indexed by the value of* λ, $\lambda > 0$. *Then,* (*i*) $\lim_{\lambda\to\infty} E_\lambda = 0$, $\forall a > a^*$, *and* (*ii*) $\lim_{\lambda\to\infty} \widehat{a}_T(\lambda) = a^*$.

Proof: We start out by showing that $\forall a > a^* + \epsilon$, $\epsilon > 0$, $\lim_{\lambda\to\infty} E_\lambda(a, \widehat{v}(\lambda)) = 0$. Recall that, by equation (4.6), $E_\lambda(\widehat{a}_T(\lambda), \widehat{v}(\lambda)) = \Delta(\widehat{a}_T(\lambda))$. Hence, $h(\widehat{v}(\lambda)) = \Delta(\widehat{a}_T(\lambda))g(\widehat{a}_T(\lambda))^{-\lambda}$ and for all $a > \widehat{a}$, $E_\lambda(a, \widehat{v}(\lambda)) = \Delta(\widehat{a}_T(\lambda)) \left[\frac{g(\widehat{a}_T)}{g(a)}\right]^\lambda$. Note that $\Delta(\widehat{a}_T(\lambda))$ is bounded above by $\Delta(a^*)$ and below by $\Delta(\widehat{a}_M)$ (where $\widehat{a}_M$, as defined in equations (4.1) and (4.2), is the lowest ability level that attends H under markets, and hence $\Delta(\widehat{a}_T(\lambda) \geq \Delta(\widehat{a}_M)$ $\forall\lambda$). Note furthermore that $\forall a > a^* + \epsilon$, $\epsilon > 0$, $\frac{g(\widehat{a}_T)}{g(a)}$ is smaller than one and is bounded from above by $\frac{g(a^*)}{g(a^*+\epsilon)}$. Hence, $\lim_{\lambda\to\infty} E_\lambda(a, \widehat{v}(\lambda)) = 0$, $\forall a > a^* + \epsilon$.

Note that the above implies that all individuals with $a > a^* + \epsilon$ attend H, since all can afford to and furthermore prefer it to L. But, since this conclusion holds for all $\epsilon > 0$, it follows that $\lim_{\lambda\to\infty}\widehat{a}_T(\lambda) = a^*$.||

Corollary 4.9. $\lim_{\lambda\to\infty} \widehat{C}_T(\lambda) = C^* > \widehat{C}_M$.

Proof: The preceding theorem established that as λ tends to ∞, the allocation of individuals to school tends to the efficient allocation and test expenditures tend to zero. Hence aggregate consumption tends to its first best level, C^*.||

The above theorem establishes that with a sufficiently powerful test technology exams dominate markets both in terms of matching efficiency and in terms of aggregate consumption. In the limit, matching is efficient, guaranteeing that output is maximized. Furthermore, in the limit waste is zero, so aggregate consumption is also at its maximum.

The intuition for the above result comes from noting that an increase in λ increases E_a. Noting that $E_a = -\frac{V_a}{V_e}$, another way of saying the same thing is to state that an increase in λ increases the importance of ability relative to expenditures. That is, ceteris paribus, an increase in λ makes it easier for higher ability but low wealth individuals to compete with lower ability but higher wealth individuals with respect to admission to a high quality school. And, precisely because, ceteris paribus, a greater proportion of the competition can come through ability than through expenditures, waste may be reduced.[17]

It should be noted that the opposite result to that of Corollary 4.9 is obtained if we instead take the limit as $\lambda \to 0$. This makes ability totally irrelevant to the score obtained, i.e. only expenditures matter. In that case, the allocation achieved under exams is the same as under markets. Under exams, wasteful spending of the amount $(1-\alpha)\widehat{P}_H$ occurs, however, guaranteeing that while output is the same under both mechanisms, exams achieve lower aggregate consumption.

5. An Example

In this section we provide an example that illustrates how exams and markets fare as allocation mechanisms. We consider the following case: Individuals are distributed uniformly (i.e. a and w are uniformly distributed); $X(a,s) = as$, hence $\Delta(a) = az$, $z \equiv H - L$; and, lastly, $E(a,v) = h(v)a^{-\lambda}$ (with $h, h' > 0$).

We start out by contrasting the equilibrium achieved under both mechanisms with perfect capital markets. Note that the uniform distribution of individuals implies $a^* = \alpha$.

Applying equation (3.9), we obtain:

$$\begin{aligned} Y^* &= \frac{1}{\overline{w}}\int_{a^*}^{1}\int_{0}^{\overline{w}} aH\ dwda + \frac{1}{\overline{w}}\int_{0}^{a^*}\int_{0}^{\overline{w}} aL\ dwda \\ &= \frac{H(1-\alpha^2)}{2} + \frac{L\alpha^2}{2} \end{aligned} \tag{5.1}$$

[17] The reason for the ceteris paribus qualifier in the statement is that a more powerful technology will increase the value of $\widehat{a}_T$ but this will also increase the expenditures of this agent and of agents with similar ability levels. Thus, there is no reason to believe that waste is monotonically decreasing in λ.

and thus, noting that $W = \frac{\overline{w}}{2}$,

$$C_M^* = \frac{H(1-\alpha^2)}{2} + \frac{L\alpha^2}{2} + \frac{\overline{w}}{2} \tag{5.2}$$

To calculate consumption under exams, note that $\Delta(\alpha) = E(\alpha, v^\star)$, yielding $h(v^\star) = z\alpha^{1+\lambda}$. Thus,

$$\varepsilon^\star = \frac{1}{\overline{w}} \int_{a^*}^{1} \int_{0}^{\overline{w}} h(v^\star) a^{-\lambda} \; dwda = \frac{z\alpha^{1+\lambda}(1-\alpha^{1-\lambda})}{1-\lambda} \tag{5.3}$$

and

$$C_T^* = Y_T^* - \varepsilon^* + W = \frac{H(1-\alpha^2)}{2} + \frac{L\alpha^2}{2} - \frac{z\alpha^{1+\lambda}(1-\alpha^{1-\lambda})}{1-\lambda} + \frac{\overline{w}}{2} \tag{5.4}$$

With borrowing constraints, using equilibrium conditions (4.1) and (4.2), yields:

$$\widehat{a}_M = \frac{(z+\overline{w}) - \sqrt{(z+\overline{w})^2 - 4z\alpha\overline{w}}}{2z} \tag{5.5}$$

and using equation (4.4) gives:

$$\widehat{Y}_M = \frac{1}{2\overline{w}} [H(1-\widehat{a}_M^2)\overline{w} + L\widehat{a}_M^2(\overline{w} - z\widehat{a}_M)] \tag{5.6}$$

with $\widehat{C}_M = \widehat{Y}_M + \frac{\overline{w}}{2}$.

Under exams, applying equations (4.6) and (4.7) we obtain an implicit expression for $\widehat{a}_T$:

$$\frac{1}{\overline{w}} \int_{\widehat{a}_T}^{1} \int_{z\widehat{a}_T \widehat{\left(\frac{a_T}{a}\right)}^{\lambda}}^{\overline{w}} dw \; da = 1 - \alpha \tag{5.7}$$

and, using equations (4.9) and (4.11), yields:

$$\widehat{Y}_T = \frac{H(1-\widehat{a}_T^2)}{2} + \frac{zH\widehat{a}_T^{1+\lambda}(1-\widehat{a}_T^{2-\lambda})}{\overline{w}(2-\lambda)} + \frac{L\widehat{a}_T^2}{2} + \frac{zL\widehat{a}_T^{1+\lambda}(1-\widehat{a}_T^{2-\lambda})}{\overline{w}(2-\lambda)} \tag{5.8}$$

$$\widehat{\varepsilon} = \frac{z\widehat{a}_T^{1+\lambda}(1-\widehat{a}_T^{1-\lambda})}{1-\lambda} - \frac{z^2\widehat{a}_T^{2(1+\lambda)}(1-\widehat{a}_T^{1-2\lambda})}{(1-2\lambda)\overline{w}} \tag{5.9}$$

with $\widehat{C}_T$ given by $\widehat{Y}_T - \widehat{\varepsilon} + \frac{\overline{w}}{2}$.

Table 1 reports the equilibrium values of a^*, output, consumption, and in the case of exams, waste, with perfect capital markets for different values of λ. Note that, of course, the values of output and a^* are invariant to the value of λ since the efficient allocation (and hence output) is independent of the test technology. Consumption under markets is likewise independent of λ since outcomes under markets do not depend on the test technology. With exams, note that the value of waste, $\varepsilon^\star$, and hence consumption, $c^\star$, are affected by λ, with waste decreasing, and hence consumption increasing, for greater values of λ.

Table 2 reports the values of $\widehat{a}$, output, consumption, and waste under borrowing constraints for the same values of λ as in Table 1. Note that for λ relatively small, markets dominate exams in terms of aggregate consumption. At an intermediate value of λ ($\lambda = 5.309$), both mechanisms yield the same consumption level. And, for larger values of λ, aggregate consumption is greater under exams than under markets.

It is interesting to note that, as indicated by the values of consumption in the tables below, exams under borrowing constraints can dominate exams under perfect capital markets in terms of aggregate consumption, even while being inferior to markets with borrowing constraints.[18]

Table 1*
Perfect Capital Markets

	λ	$a^\star$	$y^\star$	$c^\star - W$	$\varepsilon^\star$
Markets	-	0.5	1.25	1.25	-
Exams	0.2	0.5	1.25	0.7868	0.4632
	3	0.5	1.25	1.0625	0.1875
	5.309	0.5	1.25	1.1398	0.1102
	10	0.5	1.25	1.1946	0.0554
	50	0.5	1.25	1.2398	0.0102

[18] For the linear production technology chosen here, we found that consumption, waste, and output were all monotonic functions of λ and that consumption under exams with borrowing constraints was always higher than with perfect capital markets. In general, this need not be the case.

Table 2*
Borrowing Constraints

	λ	$\widehat{a}$	$\widehat{y}$	$\widehat{c}-W$	$\widehat{\varepsilon}$
Markets	-	0.3486	1.1602	1.1602	-
Exams	0.2	0.3628	1.1843	0.8617	0.3226
	3	0.4467	1.2346	1.0982	0.1364
	5.309	0.4675	1.2436	1.1602	0.0834
	10	0.4828	1.2482	1.2044	0.0438
	50	0.4966	1.2499	1.2415	0.0084

*The calculations above are for $H = 3$, $L = 1$ (so $z = 2$), $\overline{w} = 3$,and $\alpha = 0.5$. In order to allow easier comparisons between output and consumption, we have subtracted aggregate wealth from consumption.

6. Other Mechanisms

In this section we examine the implications of different allocative mechanisms all under the assumption that the market for loans is inoperative.[19]

6.1. Taxation

A very simple mechanism that ensures matching efficiency is to redistribute first-stage wealth (endowments) so that all individuals end up with equal endowments.[20] This extreme scheme guarantees that wealth does not matter to the final allocation since the market (or for that matter, the test technology) can no longer discriminate among individuals based on their ability to pay, but rather solely on their willingness to pay. There are several (correct) objections, however, that can be raised to such a proposal. First, in the absence of a dynamic model,

[19] We have chosen not to specify the particular mechanism that closes down borrowing against human capital. This has the benefit of not introducing additional assumptions or details of the model that do not bear directly on the main point of this paper. The corresponding drawback is an inability to rigorously discuss interventions such as subsidized loans or other policies that would be likely to interact with the microfoundations for the inability to borrow.

[20] Complete redistribution is not necessary, in general, to obtain the first best allocation. It is sufficient to ensure that all agents can afford P_H^*. Of course, the latter may not be feasible (i.e., W may be smaller than P_H^*).

taxation of wealth erroneously appears to be an entirely innocuous instrument; it causes no distortions and hence maximizes aggregate consumption. In a more sophisticated model, with savings and effort decisions, this would not be the case. Second, if wealth is not observable, how is the government to tax it effectively?

What we consider here instead is the effect of taxing income obtained after schooling. This tax will have incentive effects, as will be seen shortly, but in a direction that improves matching efficiency. It will, again artificially, not have other disincentive effects due to the simple and static nature of the model (e.g. no disutility from labor, etc.). Thus, one way to think about our results comparing aggregate output and consumption under a tax policy relative to a no-tax policy, is by noting how large the potential distortions due to taxation would have to be in order to outweigh its benefits. Given these caveats, we do not solve for the optimal tax schedule, but instead restrict ourselves to examining the qualitative effects of a proportional tax, t, on income.[21]

The effect of a proportional tax, as shown in Figure 7, is to decrease the net gain from obtaining a high-quality education from $\Delta(a)$ to $(1-t)\Delta(a) \equiv (1-t)[X(a,H) - X(a,L)]$. Consequently, equilibrium condition (4.1) must be modified to:

$$(1-t)\Delta(\widehat{a}) = \widehat{P}_H \tag{6.1}$$

with (4.2) and (4.3) remaining unchanged.

The effect of this policy is straightforward to describe. The decreased attractiveness of higher-quality education lowers the demand for the latter, decreasing its price and thus making it more affordable to higher-ability but poorer individuals but still less attractive to lower-ability individuals. Consequently, as shown in Figure 7, the equilibrium value of $\widehat{a}$ increases and $\widehat{P}_H$ decreases. Output is unambiguously higher as is, in the absence of other distortions, aggregate consumption.

The effect of taxation under an exam mechanism is similar to its effect under market prices. With an exam mechanism, though, taxation has the additional benefit of decreasing wasteful expenditures, both because it decreases the spread of ability levels of the individuals that attend H, and hence the total level of expenditures associated with any given score, but also because the score associated

[21] A proportional tax on all income would be unlikely to be optimal since a similar outcome could be obtained without taxing those individuals who attend L. This would possess the same positive matching incentives as a proportional tax on all individuals, but would have the additional benefit of allowing a lower tax on H to obtain the same effects and no tax on those who attend L.

with H falls.

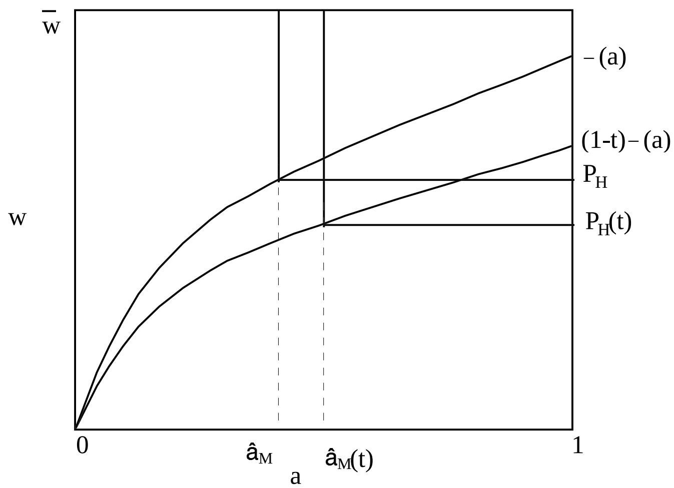

Figure 7

6.2. A Fellowship Scheme[22]

Suppose now that initial endowments of wealth are perfectly observable (but that it is still impossible or very costly to transfer these among individuals). As remarked upon earlier, the operation of markets or exams in their pure form is not affected by the observability of endowments. A possibility now open to the government, however, is to maintain the market mechanism and to give out "fellowships" on the basis of wealth and exam results.

We model the introduction of a fellowship scheme in the following way. In the first stage, the government announces the fellowship rule. This consists of an exam and a score v_f that must be achieved in order to obtain a fellowship amount $m(w)$. In the next stage, schools set prices and individuals choose whether to take the exam or to simply pay the price associated with either the high or low quality school.

The fellowship scheme we consider comes arbitrarily close to achieving the first best level of output and consumption: In equilibrium the high quality schools charge a price arbitrarily close to the price that would be obtained in equilibrium under perfect capital markets (i.e. P_H^*), the assignment of individuals to schools

[22] I thank Sherwin Rosen for suggesting that a mixed mechanism that combined both prices and exams might be optimal.

is arbitrarily close to the first best assignment, and wasteful expenditures are arbitrarily close to zero.

Below we describe the properties of the fellowship mechanism. For expositional simplicity we consider the case where $P_H^* \leq \overline{w}$, i.e., $\Delta(a^*) \leq \overline{w}$.[23] Let the government's fellowship rule be given by:

$$v_f = V(a^\star, \epsilon) \tag{6.2}$$

i.e., the cutoff score for a fellowship is the score that would be obtained by an individual of ability a^* who incurred test expenditures of ϵ, $\epsilon > 0$. All individuals who obtain v_f and whose wealth lies below P_H are awarded a fellowship of the amount:

$$m(w) = P_H - w + \epsilon \tag{6.3}$$

Note that this fellowship rule awards ϵ plus the difference between the price of the higher-quality school and the individual's wealth to all those who obtain the fellowship score and whose wealth is lower than P_H.

When faced with this rule, all individuals with $\min(\Delta(a), w) \geq P_H$ will be willing and able to afford to attend the high-quality school, and will pay the high-quality school's price in order to do so.[24] Furthermore, all individuals with $a \geq a^*$ and $w - E(a, v_f) + \epsilon \leq \min(P_H, \Delta(a))$ will also desire to attend the high-quality school and will obtain a fellowship in order to do so. Note that most of these individuals would be able to afford to attend H with a lower fellowship amount than $m(w)$ since to obtain v_f they incur a cost of $E(a, v_f)$ which is strictly smaller than ϵ for all individuals with ability greater than a^*. If exam expenditures are unobservable (a maintained assumption throughout), however, then the government is unable to keep the difference between ϵ and $E(a, v_f)$. Of course, individuals with $a < a^*$ and $w - E(a, v_f) + \epsilon \leq \min(P_H, \Delta(a))$ also desire a fellowship to attend H, but the fellowship amount is designed to be insufficient to allow them to do so since they would have to spend more than ϵ in order to obtain v_f, leaving them with insufficient funds to afford P_H.

[23] The sole modification that must be made if $P_H^* > \overline{w}$ is for the government to place an upper bound on the fellowship that it will pay of $P_H^* - w + \epsilon$, otherwise schools have an incentive to charge a higher price knowing that the government, rather than individuals, will pay the incremental cost.

[24] For the same reasons as described for the other mechanisms, $P_L = 0$. That is, in equilibrium there will be a measure of agents who attend L whose wealth is arbitrarily close to zero, requiring a price of zero for the lower-quality school in order to clear markets.

Note that the above discussion implies that in equilibrium P_H will be strictly smaller than P_H^* since if the price were at its perfect capital market level, the fraction of individuals willing and able to attend with this fellowship scheme would be smaller than $1-\alpha$. Thus, in equilibrium we must have

$$P_H = \Delta(a') \tag{6.4}$$

and

$$\int_{E(a,v_f)}^{\overline{w}} \int_{a^*}^{1} f(a,w)\ da\ dw + \int_{P_H}^{\overline{w}} \int_{a'}^{1} f(a,w)\ da\ dw = 1-\alpha \tag{6.5}$$

Thus,

$$S(a,w) = \begin{cases} H,\ \forall a \geq a^* \text{ and } w \geq E(a,v_f) \\ H,\ \forall a \in [a',a^*) \text{ and } w \geq P_H \\ L, \text{ otherwise} \end{cases} \tag{6.6}$$

Furthermore, exam expenditures by each agent, $e(a,w)$, are given by:

$$e(a,w) = \begin{cases} E(a,v_f),\ \forall\ a \geq a^* \text{ and } w \in [E(a,v_f), P_H) \\ 0, \text{ otherwise} \end{cases} \tag{6.7}$$

yielding total waste of:

$$\varepsilon = \int_{a^*}^{1} \int_{E(a,v_f)}^{\Delta(a')} E(a,v_f) f(a,w)\ dwda \tag{6.8}$$

Describing the above in words, in equilibrium all agents with $a \geq a^*$ who can afford it (either by paying or through fellowships) attend H, i.e.e, all those with $w \geq E(a,v_f)$ (note that the upper bound for $E(a,v_f)$ for these individuals is ϵ). In addition, those agents with $a \in [a',a^*)$, and who can afford to attend without obtaining a fellowship will also attend H. All other agents attend L. This is depicted in Figure 8 where the diagonally shaded area illustrates the set of agents that attend H by paying the market price for it and the vertically shaded area represents those that attend H by qualifying for a fellowship.

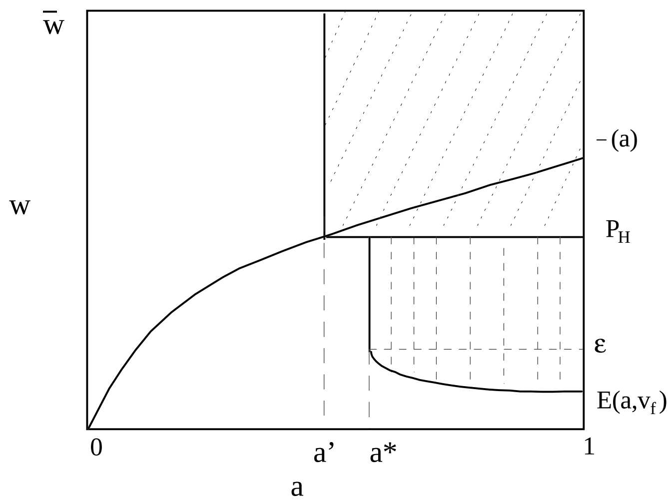

Figure 8

Note that the smaller is ϵ, the closer a' is to a^* and that the distance between these two can be made arbitrarily small by making ϵ sufficiently small. Hence, as $\epsilon \longrightarrow 0$, the allocation tends to the efficient one and $P_H \longrightarrow P_H^*$. Furthermore, as $\epsilon \longrightarrow 0$, v_f does likewise, implying that aggregate waste also tends to zero. Thus, for a small enough ϵ, the allocation of individuals to schools (and hence output), and aggregate consumption, come arbitrarily close to the first best level, Y^* and C^*.[25]

It is interesting, and perhaps counterintuitive, to observe that the above fellowship scheme which maximizes both output and aggregate consumption does not require agents to obtain the greatest score possible compatible with their wealth and some minimum ability level, say a^*. In fact, the score that they are required to obtain is independent of their wealth, despite the fact that the latter is observable. To see why, consider a fellowship scheme that requires individuals to obtain the score $\overline{v}_f(w)$ defined by $E(a^\star, \overline{v}_f(w)) = w$ in order to obtain a fellowship of $m(w) = P_H^* - E(a^*, \overline{v}_f(w))$ for $w < P_H^*$. That is, individuals with wealth w are required to obtain the score that an individual with ability a^* would obtain by spending w. Such a scheme will indeed deliver the first-best allocation of individuals to schools. However, unlike the previous fellowship mechanism, this arrangement will not maximize aggregate consumption as well. This is due to the

[25] We are once again abstracting from the costs that would be incurred by the necessity of raising funds for these fellowships.

large amount of wasteful expenditures that such a high score requires, and which must be incurred by all high ability individuals unable to afford the high-quality school. Recall that whereas in the preceding fellowship scheme individuals were spending ϵ and below, under this alternative setup individuals whose wealth lies below P_H^* must spend close to their entire wealth generating the fellowship score. Furthermore, it is also interesting to note that this scheme requires the government to raise a larger quantity of funds since individuals receive as fellowships the difference between P_H^* and $w - E(a, \overline{v}_f(w))$ (which recall for $a = a^*$ equals a fellowship of P_H^*) rather than (at most) the difference between P_H^* and $w - E(a, v_f)$ (which recall for $a = a^*$ equals a fellowship smaller than $P_H^* + \epsilon - w$).

7. Conclusion

This paper examined the properties of exams and markets as alternative allocation devices under borrowing constraints. We find that exams dominate markets in terms of matching efficiency. Whether aggregate consumption is greater under exams than under markets, however, depends on the power of the exam technology; for a sufficiently powerful test, exams dominate markets in terms of aggregate consumption as well. We analyze as well the effects of income taxation and characterize the optimal allocation scheme when wealth is observable is derived. The latter consists of a fellowship scheme in which markets set school prices but the government gives out fellowships based on need and the ability to obtain a given exam score.

This paper can be considered a first step in thinking about why countries use different mixes of prices and exams to guide admission decisions. For example, if wealth (or income) is easily observable (something that one may think more likely in an advanced economy with a sophisticated taxing and auditing system than in a less developed country), then a fellowship scheme based on need and exam scores will tend to be a more efficient allocation mechanism than either prices or exams used on their own. Thus, ceteris paribus, one might expect to see developed countries relying more on fellowship schemes and prices (with private ownership of schools) and developing countries relying more on exams and public ownership of schools. The latter to a large extent is true, with most developing countries characterized by public universities, low fees, and exams to guide admission decisions. The characterization of advanced economies, however, is more heterogenous, with some countries such as Japan relying on exams at virtually every stage of the educational process whereas others use a mixture of exams and prices. Similarly, one might expect to see countries in which credit constraints against human capital are less binding relying on a price rather than an exam mechanism.

There are many additional questions that suggest themselves for future research. One potential drawback of the analysis presented is that the distribution of schools is exogenous. One might very well expect that their distribution, however, would depend on the allocation mechanism used. One problem with endogenizing the school distribution lies with the specification of the objective function of schools. Most schools are non-profit institutions with a variety of actors with potentially quite different objectives playing important roles in decision making, e.g. principles, trustees, teachers, parents, and local and state government.[26] Other avenues for future research include incorporating peer effects into the analysis and thinking about how different levels of education may call for different allocation mechanisms. If, for example, human capital accumulation prior to tertiary level is important in determining future outcomes, then what may matter most to the efficiency of an education system is that access to high quality primary and secondary education be independent of income (though not necessarily independently of ability). Lastly, there are undoubtedly questions of political economy that guide the use of different allocation schemes as well as considerations of efficiency. Note, for example, that individuals that attend high quality schools in the presence of borrowing constraints are actually better off than with perfect capital markets since borrowing constraints lower the price of the high quality school or the score required for admission. Similarly, there will be a group of agents that prefers a less powerful test technology than a more powerful one since the former allows wealth to play a greater role relative to ability.

[26] See Rothschild and White (1993) for an insightful discussion of the application of economic analysis to the study of university behavior.

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